MCS 108,HW10

Q1. A delivery company accepts only rectangular boxes where $x + 2y + 2z = 108$. Find the dimensions of an acceptable box of largest volume.

Q2. Using Lagrange Multipliers method, find the extreme values of $f(x, y) = 2x^2 + y + y^2$ on the circle $x^2 + y^2 = 1$.

Q3. Find the absolute maximum of $f(x, y) = x^2 + xy - y^2 - 5x$ on the region bounded by the coordinate axes and the line $x + y = 4$.

Q4. Using Lagrange Multipliers find the minimum of $f(x, y, z) = x - 2y + 2z$ subject to the constraint $x^2 + y^2 + z^2 = 9$.

Q5. Using Lagrange Multipliers find the maximum value of $f(x, y, z) = xyz$ subject to the constraint $x + 2y + 3z = 36$.

Q6. Using Lagrange Multipliers find the minimum value(s) of $f(x, y) = 2x^2 + xy + y^2 + 200$ subject to the constraint $x + y = 200$.

Q7. Using Lagrange Multipliers find the extreme value(s) of
a) $f(x, y) = x^2 + 4y^2 + 6$ subject to the constraint $2x - 8y = 20$.
b) $f(x, y) = 3x^2 - 2y^2 + 9$ subject to the constraint $x + y = 1$.
c) $f(x, y, z) = xyz^2$ subject to the constraint $x - y + z = 20$.

Q8. Using Lagrange Multipliers find the minimum value(s) of $f(x, y, z) = x^2 + y^2 + z^2$ subject to the constraint $x + y + z = 3$.

Q9. The production function for a firm is $f(p, q) = 20p + 25q - p^2 - 3q^2$ The cost to the firm of p and q is 2 and 4 respectively. If the firm wants the total cost of input to be 50, find the greatest output possible, subject to this budget constraint. That means, Using Lagrange Multipliers find the maximum value of $f(p, q)$ subject to the constraint $2p + 4q = 50$.

Q10. Using Lagrange Multipliers find the minimum value(s) of $f(x, y, z) = x + y + z$ subject to the constraint $xyz = 8$.

Q11. Find and classify all critical points of the function $f(x, y) = x^2 + 4y^2 - 6x + 16y$.

Q12. Find and classify all critical points of the function $f(x, y) = xy - x + y$.

Q13. Let P be a production function given by

$$P = f(l, k) = 0.54l^2 - 0.02l^3 + 1.89k^2 - 0.09k^3$$

where l and k are the amounts of labor and capital, respectively , and P is the quantity of output produced. Find the values of l and k that maximize P.

Q14. Find and classify all critical points of the function $f(x, y) = 2x^2 + y^2 - 2xy + 5x - 3y + 1$.

Q15. Find the indicated derivatives by using the chain rule.
a) $w = 2x^2 + 3xy + 2y^2, x = r^2 - s^2, y = 4r^2 + s^2$; $w_r(1, 1) =?$ and $w_s(1, 1) =?$
b) $w = x^2yz + xy^2z + xyz^2, x = e^t, y = te^t, z = t^2e^t$; $w_t =?$
c) $w = \ln(xyz), x = r^2s, y = rs, z = rs^2$; $w_r =?$ and $w_s =?$.

Q16. Suppose the cost C of producing q_A units of product A and q_B units of product B is given by

$$C = \sqrt[3]{3q_A^2 + q_B^3 + 4}$$

and the demand functions are given by $q_A = 10 - p_A + p_B^2$ and $q_B = 20 + p_A - 11p_B$. Evaluate C_{p_A} and C_{p_B}.