

Exam

Name _____

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Provide an appropriate response.

1) By using matrix reduction, solve the system:

$$\begin{cases} x_1 + 2x_2 - 3x_3 + x_4 = 1 \\ 2x_1 + x_2 + 2x_4 = 8 \end{cases}$$

1) _____

2) Without solving, determine whether the following system of linear equations has a unique solution, or infinitely many solutions.

$$\begin{cases} 3x + 5y - 11z = 0 \\ 9x + 13y - 15z = 0 \end{cases}$$

2) _____

3) Solve the following system of equations by reduction.

$$\begin{cases} 2x + 3y + 4z + 9u = 4 \\ 3x + 4y + 8z + 7u = 1 \end{cases}$$

3) _____

A plane in three dimensional space can be written as $ax + by + cz = d$. We can find the possible intersections of planes in this form by writing them as systems of linear equations and using reduction to solve them. If $d = 0$ in each equation, then we have a homogeneous system with either a unique solution or infinitely many solutions.

4) Determine whether the intersection of the planes:
$$\begin{cases} 3x - 2y - z = 0 \\ x + 3y - 5z = 0 \\ 2x - 7y + 3z = 0 \end{cases}$$

4) _____

has a unique solution or infinitely many solutions; then solve the system.

5) Determine whether the intersection of the planes:
$$\begin{cases} 5x + 2y + z = 0 \\ 2x - y + 3z = 0 \\ 3x - 2y + 6z = 0 \end{cases}$$

5) _____

has a unique solution or infinitely many solutions; then solve the system.

6) Determine whether the intersection of the planes:
$$\begin{cases} 2x + y + z = 0 \\ 5x + 4y + 5z = 0 \\ x + 2y + 3z = 0 \end{cases}$$

6) _____

has a unique solution or infinitely many solutions; then solve the system.

Provide an appropriate response.

7)

If $A = \begin{bmatrix} 1 & 0 & 3 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$, use row reduction to determine A^{-1} providing it exists.

7) _____

8) (a) If A is the coefficient matrix of the system $\begin{cases} x + 3y = 2 \\ x + 2y = 5 \end{cases}$, determine A^{-1} .

8) _____

(b) Use A^{-1} to solve the system.

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

9)

9) _____

If $R = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 2 & -1 \\ 1 & 1 & 0 \end{bmatrix}$, then $R^{-1} =$

A) $\begin{bmatrix} 1 & 0 & 1 \\ -1 & 0 & 0 \\ -1 & -1 & 1 \end{bmatrix}$

B) $\begin{bmatrix} \frac{3}{2} & \frac{5}{2} & -\frac{3}{2} \\ -1 & \frac{4}{3} & \frac{1}{2} \\ 2 & \frac{2}{3} & -\frac{5}{3} \end{bmatrix}$

C) $\begin{bmatrix} 2 & -1 & 2 \\ -1 & 1 & 3 \\ 4 & 2 & -2 \end{bmatrix}$

D) $\begin{bmatrix} 1 & 0 & -3 \\ -1 & 2 & 4 \\ 0 & 1 & 6 \end{bmatrix}$

E) none of the above

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

10) Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$; find A^{-1} .

10) _____

11) Look at the equations $\begin{cases} 2x + y = 5 \\ 7x + 4y = 7 \end{cases}$

11) _____

(a) Set up these equations in the matrix form $Ax = b$

(b) Find A^{-1}

(c) Using A^{-1} , solve the equations.

12)

12) _____

Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -3 & -6 & -10 \end{bmatrix}$. Find A^{-1} .

13) (a) If A is the coefficient matrix of the system $\begin{cases} x + 2y + 3z = 1 \\ 2x + 5y + 7z = 9 \\ -3x - 6y - 10z = 8 \end{cases}$, determine A^{-1} .

13) _____

(b) Use A^{-1} to solve the system.

14) Find the inverse matrix of $\begin{bmatrix} 2 & 1 \\ -2 & 3 \end{bmatrix}$.

14) _____

15) Find the inverse matrix of $\begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}$.

15) _____

16) Find the inverse matrix of $\begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 2 \\ 0 & 0 & -1 \end{bmatrix}$.

16) _____

Answer Key

Testname: UNTITLED3

1) $x_1 = 5 - x_3 - x_4, x_2 = -2 + 2x_3, x_3 = x_3, x_4 = x_4$

2) infinitely many solutions

3)

$$\begin{cases} x = -13 - 8z + 15u \\ y = 10 + 4z - 13u \end{cases}$$

4) a unique solution; $x = 0, y = 0, z = 0$

5) a unique solution; $x = 0, y = 0, z = 0$

6) infinitely many solutions; $x - \frac{1}{3}z = 0, y + 1\frac{2}{3}z = 0$; in parametric form: $x = \frac{1}{3}r, y = -1\frac{2}{3}r, z = r$, where r is any real

number.

7)
$$\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & 1 \\ \frac{1}{6} & \frac{1}{6} & \frac{2}{3} \\ \frac{1}{6} & \frac{1}{6} & -\frac{1}{3} \end{bmatrix}$$

8) (a)
$$\begin{bmatrix} -2 & 3 \\ 1 & -1 \end{bmatrix}$$

(b) $x = 11, y = -3$

9) A

10)
$$A^{-1} = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$$

11) (a)
$$\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$$

(b)
$$\begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}$$

(c) $x = 13, y = -21$

12)
$$A^{-1} = \begin{bmatrix} 8 & -2 & 1 \\ 1 & 1 & 1 \\ -3 & 0 & -1 \end{bmatrix}$$

13) (a)
$$A^{-1} = \begin{bmatrix} 8 & -2 & 1 \\ 1 & 1 & 1 \\ -3 & 0 & -1 \end{bmatrix},$$

(b) $x = -2, y = 18, z = -11$

14) Inverse matrix is
$$\begin{bmatrix} \frac{3}{8} & -\frac{1}{8} \\ \frac{1}{4} & \frac{1}{4} \end{bmatrix}.$$

15) Inverse matrix is
$$\begin{bmatrix} \frac{2}{5} & -\frac{1}{5} \\ \frac{3}{10} & \frac{1}{10} \end{bmatrix}.$$

Answer Key

Testname: UNTITLED3

16) Inverse matrix is $\begin{bmatrix} \frac{1}{5} & \frac{2}{5} & \frac{4}{5} \\ -\frac{2}{5} & \frac{1}{5} & \frac{2}{5} \\ 0 & 0 & -1 \end{bmatrix}$.