MCS 108 Homework 4

Q1. Using row reduction,find all solutions of the systems:
a)

$$\begin{aligned} x_1 + 2x_2 &= 1 \\ 2x_1 + x_3 &= 2 \\ 3x_1 + 2x_2 + x_3 - x_4 &= 4 \end{aligned}$$

b)

$$\begin{aligned} 2x_1 + x_2 - x_3 &= 2 \\ 2x_1 + x_3 &= 3 \\ 3_1 - x_2 &= 0 \end{aligned}$$

c)

$$\begin{aligned} x_1 + x_2 - x_3 &= 2 \\ x_1 + x_3 &= 3 \\ 2x_1 + x_2 - 2x_3 &= 0 \\ x_1 + 5x_2 + 5x_3 &= 8 \end{aligned}$$

Q2. Find the inverse of the matrix A= $\begin{bmatrix} 1 & 1 & 0 \\ 2 & -2 & 1 \\ 3 & 2 & -1 \end{bmatrix}$.

Q3. If

$$\mathrm{A}^{-1} = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 4 & 2 \\ 0 & 1 & -2 \end{bmatrix}, B^{-1} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ -2 & 3 & 2 \end{bmatrix}$$ *compute* (AB)$^{-1}$.

Q4. Solve the non-linear system

$$\begin{aligned} \frac{-1}{x} + \frac{3}{y} - \frac{4}{z} &= 2 \\ \frac{2}{x} + \frac{5}{y} + \frac{1}{z} &= -1 \\ \frac{1}{x} + \frac{4}{y} + \frac{4}{z} &= 3 \end{aligned}$$

(Hint: Let $u = \frac{1}{x}, v = \frac{1}{y}, w = \frac{1}{z}$)

Q5. Find coefficients a, b, c, d so that the curve $y = ax^3 + bx^2 + cx + d$ passing through the points $(0, 3), (1, 3), (2, 7), (-1, 1)$.

Q6. Evaluate the following determinants :

a) det A= $\begin{vmatrix} 3 & -4 \\ -2 & 2 \end{vmatrix}$ $b)$ $detB = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$ $c)$ $detC = \begin{vmatrix} 0 & 2 & -3 \\ 2 & 1 & 3 \\ 1 & 1 & 5 \end{vmatrix}$

Q7. Let A be 3×3 matrix and $detA = 3$. Compute

a) det (A^{-1}) , b) det $(3A)$

Q8. Evaluate the following determinants by **inspection**:

a) det A= $\begin{vmatrix} -3 & 1 & 5 \\ 1 & 7 & 9 \\ -3 & 1 & 5 \end{vmatrix}$ $b)$ $detB = \begin{vmatrix} 4 & 2 & -2 \\ -2 & -1 & 1 \\ 5 & 6 & 1 \end{vmatrix}$ $c)$ $detC = \begin{vmatrix} 4 & 6 & -3 \\ 0 & -5 & 6 \\ 0 & 0 & 0 \end{vmatrix}$

Q9.Find $AdjA$, if A= $\begin{bmatrix} 1 & 2 & -1 \\ 3 & 4 & 2 \\ 0 & 1 & -2 \end{bmatrix}$

Q10.Find the Adjoint of the following matrices

a) A= $\begin{bmatrix} 2 & -5 \\ 3 & 4 \end{bmatrix}$ $b)B = \begin{bmatrix} 2 & -5 & 0 \\ -1 & 3 & 1 \\ 0 & -1 & 2 \end{bmatrix}$ $c)C = \begin{bmatrix} 1 & -2 & 1 \\ -1 & 0 & 2 \\ 0 & 1 & 0 \end{bmatrix}$

Q11. Find the inverse of the matrices a) A= $\begin{bmatrix} 3 & 5 \\ 2 & 1 \end{bmatrix}$ $b)B = \begin{bmatrix} -1 & 3 & 0 \\ 1 & 2 & 1 \\ 3 & -1 & 1 \end{bmatrix}$.