

MCS 108 Homework 5

Q1. On A perform the following row-operations to get a matrix B ;

$-2R_3 \rightarrow R_3, R_1 \leftrightarrow R_4, 2R_3 + R_1 \rightarrow R_1, 3R_4 \rightarrow R_4$. If $\det B = 8$, what is $\det A$?

Q2. Let $A = \begin{bmatrix} 1 & -2 & 0 \\ -1 & 4 & 1 \\ 6 & 0 & -2 \end{bmatrix}$.

Find $\det A$ using;

- a) cofactor expansion along the third row
- b) cofactor expansion along the second row
- c) row-reduction.

Q3. If

$$A^{-1} = \begin{bmatrix} 5 & 0 & -1 \\ -3 & 3 & 2 \\ 0 & 4 & -3 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 & -1 \\ 4 & 0 & -2 \\ -2 & 1 & 1 \end{bmatrix}.$$

- a) compute $\det A$
- b) compute $\det(B^2)$
- c) compute $\det(4A^{-1})$

Q4. Find the values of x so that $\det(A) \neq 0$ for the matrix $A = \begin{vmatrix} x^3 & 4 & 1 \\ x^2 & -8 & 2 \\ 1 & 1 & -1 \end{vmatrix}$.

Q5. Evaluate the following determinants :

$$\text{a) } \det A = \begin{vmatrix} 3 & 1 & -1 & -2 \\ 0 & -2 & 1 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix} \quad \text{b) } \det B = \begin{vmatrix} 4 & 0 & -6 & 10 \\ 0 & 3 & 4 & 0 \\ -3 & -6 & 5 & 0 \\ -2 & 0 & 3 & -5 \end{vmatrix} \quad \text{c) } \det C = \begin{vmatrix} 1 & 0 & 2 & -3 & 4 \\ -4 & 0 & 2 & 1 & 3 \\ 3 & 0 & 5 & 1 & -3 \\ 6 & 7 & 4 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{vmatrix}$$

Q6. Find the inverse of $A = \begin{bmatrix} 1 & -1 & 0 \\ 4 & 5 & 1 \\ -3 & 1 & 2 \end{bmatrix}$.

Q7. Given; $A = \begin{bmatrix} 4 & -2 & 1 \\ 1 & 6 & 2 \\ -3 & 1 & 0 \end{bmatrix}$

- a) Evaluate $\det A$ using cofactor method
- b) Find $\text{Adj} A$
- c) Find A^{-1} using two different methods.

Q8. Given $A = \begin{bmatrix} 1 & -1 & 0 \\ 4 & 5 & 1 \\ 1 & -3 & 2 \end{bmatrix}$.

- a) Is it invertible? Why? b) Find $AdjA$ c) Find A^{-1} (if exists)
d) On A perform the following row-operations to get a matrix B ;
 $2R_3 + R_1 \rightarrow R_1, R_2 \leftrightarrow R_1, -3R_3 \rightarrow R_3, 5R_3 + R_2 \rightarrow R_2$. Find $detB$.