

MCS 108,HW9

Q1. Find the absolute extrema of $f(x, y) = xy(1 - x^2 - y^2)$ on the rectangular region $D = \{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq 1\}$.

Q2. Find the absolute extrema of $f(x, y) = x^2 + 3y^2 - 4x + 2y - 3$ on the rectangular region $D = \{(x, y) : 0 \leq x \leq 3, -3 \leq y \leq 1\}$.

Q3. Find the absolute extrema of $f(x, y) = xy - 2x - 5y$ over the triangular region with vertices $(0, 0), (7, 0), (7, 7)$.

Q4. Find the absolute extrema of $f(x, y) = x^2 - 4xy + y^3 + 4y$ on the square $D = \{(x, y) : 0 \leq x \leq 2, 0 \leq y \leq 2\}$.

Q5. Find w_r and w_s if $w = 7x + 2y + z^2, x = \frac{r}{s}, y = r^2 + \ln s, z = 2r$

Q6. Find w_r and w_s if $w = x^3 + y^3, x = r + 5s, y = 7r - 3s$

Q7. If $f(u, v, w)$ is differentiable and $u = x - y, v = y - z, w = z - x$, show that

$$f_x + f_y + f_z = 0$$

Q8. Find the local extreme values of the function $f(x, y) = xy - x^2 - y^2 - 2x - 2y + 4$

Q9. Find and classify all critical points of the functions

a) $f(x, y) = 5xy - 7x^2 + 3x - 6y + 2$

b) $f(x, y) = x^2 - 4xy + y^2 + 6y + 2$

c) $f(x, y) = 2x^2 + 3xy + 4y^2 - 5x + 2y$

d) $f(x, y) = x^2 - y^2 - 2x + 4y + 6$

e) $f(x, y) = x^2 + 2xy$

Q10. Find z_x and z_y if $2xy - z + e^{x+y} - z^2 = 0$ at point $P(0, \ln 2, 1)$.

Q11. Find and classify all critical points of the function $f(x, y) = 2x^3 + 3xy + 2y^3$.

Q12. Find and classify all critical points of the function $f(x, y) = x^3 + 3x^2y + y^3 - 15y^2 + 2$.

Q13. Given $x + yz - xz^2 = 0$, evaluate $\frac{\partial z}{\partial x}$.

Q14. Let $w = w(u, v), u = xy, v = x - y$.

Suppose $w_u(1, 2) = 1$, $w_v(1, 2) = 3$, $w(2, -1) = 5$, $x = 1$, $y = 2$, find $w_x(1, 2)$ and $w_y(1, 2)$.