



KEY

ÇANKAYA UNIVERSITY

Department of Mathematics and Computer Science

MCS 108 - Calculus for Business and Economics II

FIRST MIDTERM EXAMINATION

21.07.2016

STUDENT NUMBER:

NAME-SURNAME:

SIGNATURE:

INSTRUCTOR:

DURATION: 100 minutes

Question	Grade	Out of
1		20
2		18
3		24
4		16
5		25
Total		103

**IMPORTANT NOTES:**

- 1) Please make sure that you have written your student number and name above.
- 2) Check that the exam paper contains 5 problems.
- 3) Show all your work. No points will be given to correct answers without reasonable work.

1) Evaluate.

$$\text{a) (4 pnt)} \int \left( 2x^6 - x\sqrt{x} + \frac{1}{x^3} - 5 \right) dx = \frac{2x^7}{7} - \frac{x^{5/2}}{5/2} + \frac{x^{-2}}{-2} - 5x + C$$

$$\begin{aligned}\text{b) (4 pnt)} \int \frac{3x^5 + \sqrt{x} + 1}{x} dx &= \int \left( \frac{3x^5}{x} + \frac{\sqrt{x}}{x} + \frac{1}{x} \right) dx \\ &= 3\frac{x^4}{4} + \frac{x^{1/2}}{1/2} + \ln|x| + C\end{aligned}$$

$$\begin{aligned}\text{c) (6 pnt)} \int_1^2 (2x-1)(x^2-1) dx &= \int_1^2 (2x^3 - 2x - x^2 + 1) dx \\ &= \left[ \frac{2x^4}{4} - \frac{2x^2}{2} - \frac{x^3}{3} + x \right]_1^2\end{aligned}$$

$$= \left[ 8 - 4 - \frac{8}{3} + 2 \right] - \left[ \frac{1}{2} - 1 - \frac{1}{3} + 1 \right] = 6 - \frac{8}{3} - \frac{1}{6} = \frac{19}{6}$$

$$\begin{aligned}\text{d) (6 pnt)} \int \frac{2x+3}{(x^2+3x+1)^3} dx &= \int \frac{du}{u^3} = \int u^{-3} du = \frac{u^{-2}}{-2} + C \\ &= \frac{(x^2+3x+1)^{-2}}{-2} + C\end{aligned}$$

$$u = x^2 + 3x + 1$$

$$du = (2x+3)dx$$

2) Evaluate.

$$\text{a) (6 pnt)} \int e^{2x^3+x^2} (3x^2 + x) dx = \int e^u \frac{du}{2} - \frac{e^u}{2} + C = e^{2x^3+x^2} + C$$

$$u = 2x^3 + x^2$$

$$du = (6x^2 + 2x) dx$$

$$\frac{du}{2} = (3x^2 + x) dx$$

$$\text{b) (6 pnt)} \int x \sqrt{x^2 + 5} dx = \int \sqrt{u} \frac{du}{2} = \frac{\frac{u^{3/2}}{3}}{\frac{1}{2}} + C$$

$$u = x^2 + 5$$

$$du = 2x dx$$

$$= \frac{(x^2 + 5)^{3/2}}{3} + C$$

$$\text{c) (6 pnt)} \int xe^x dx$$

$$u = x \quad du = dx$$

$$dv = e^x dx \quad v = e^x$$

$$\int u dv = uv - \int v du$$

$$\int xe^x = xe^x - \int e^x dx$$

$$= xe^x - e^x + C$$

$$3) \text{ a) (8 pnt) Evaluate } \int \frac{4}{x^2 - 2x - 3} dx = \int \frac{4}{(x-3)(x+1)} dx$$

$$\frac{4}{(x-3)(x+1)} = \frac{A}{x-3} + \frac{B}{x+1}$$

$$A(x+1) + B(x-3) = 4$$

$$x=3 \quad 4A = 4 \quad A=1$$

$$x=-1 \quad -4B = 4 \quad B=-1$$

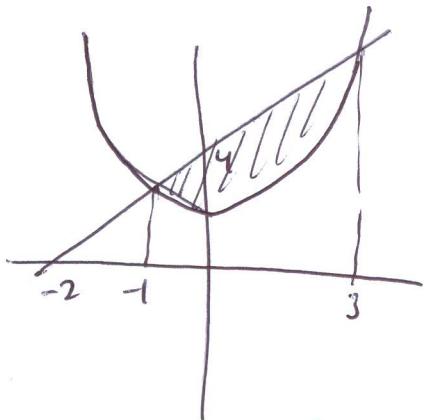
$$I = \int \left( \frac{1}{x-3} - \frac{1}{x+1} \right) dx = \ln|x-3| - \ln|x+1| + C$$

b) (8 pnt) If  $y$  is a function of  $x$  such that  $\frac{dy}{dx} = 2x + \frac{1}{\sqrt{x}}$  and  $y(0) = 4$ , find  $y(x)$ .

$$y = \int \frac{dy}{dx} dx = \int \left( 2x + \frac{1}{\sqrt{x}} \right) dx = 2 \frac{x^2}{2} + \frac{x^{1/2}}{1/2} + C = x^2 + 2\sqrt{x} + C$$

$$y(0) = 4 \quad C=4 \quad y(x) = x^2 + 2\sqrt{x} + 4$$

c) (8 pnt) Find the area of the region between the curves  $y = x^2 + 1$  and  $y = 2x + 4$ .



$$y = x^2 + 1 \quad y = 2x + 4$$

$$x^2 + 1 = 2x + 4$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$$x=3 \quad x=-1$$

$$\text{Area} = \int_{-1}^3 [2x+4 - (x^2+1)] dx = \int_{-1}^3 [2x - x^2 + 3] dx = \left[ x^2 - \frac{x^3}{3} + 3x \right]_{-1}^3$$

$$= \left[ 2^2 - \frac{2^3}{3} + 2 \cdot 2 \right] - \left[ (-1)^2 - \frac{(-1)^3}{3} + 3(-1) \right] = \frac{10}{3} + 2 - \frac{1}{3} = 9\frac{1}{3}$$

4) Consider the matrices

$$A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \\ 3 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 0 \\ 3 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 4 & -2 & 7 \\ 1 & -1 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 2 \\ -3 \end{bmatrix}.$$

a) (4 pnt) Calculate  $AB$  if it is possible. If not explain why?

$$AB = \begin{bmatrix} 1 & 2 \\ 2 & -1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ -5 & -1 \\ 0 & 1 \end{bmatrix}$$

b) (4 pnt) Calculate  $C^T$ .

$$C^T = \begin{bmatrix} 4 & 1 \\ -2 & -1 \\ 7 & 0 \end{bmatrix}$$

c) (4 pnt) Calculate  $AB + BD$  if it is possible. If not explain why?

$$BD = \begin{bmatrix} -1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ -3 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

$$AB = \begin{bmatrix} 5 & 2 \\ -5 & -1 \\ 0 & 1 \end{bmatrix} \quad \text{since the size of } AB \text{ and } BD \text{ not equal, we cannot add.}$$

d) (4 pnt) Calculate  $AB + 2C^T$  if it is possible. If not explain why?

$$AB + 2C^T = \begin{bmatrix} 5 & 2 \\ -5 & -1 \\ 0 & 1 \end{bmatrix} + 2 \begin{bmatrix} 4 & 1 \\ -2 & -1 \\ 7 & 0 \end{bmatrix} = \begin{bmatrix} 13 & 4 \\ -9 & -3 \\ 14 & 1 \end{bmatrix}$$

5) a) (13 pnt) Consider the linear system

$$\begin{aligned}x + 2y &= 1 \\x + 3y + 3z &= 2 \\2x + 3y - 2z &= -1\end{aligned}$$

(i) Form the augmented matrix of the given system.

(ii) Solve the system by using matrix reduction.

$$\left[ \begin{array}{ccc|c} 1 & 2 & 0 & 1 \\ 1 & 3 & 3 & 2 \\ 2 & 3 & -2 & -1 \end{array} \right] \xrightarrow{\begin{matrix} R_2 - R_1 \rightarrow R_2 \\ R_3 - 2R_1 \rightarrow R_3 \end{matrix}} \left[ \begin{array}{ccc|c} 1 & 2 & 0 & 1 \\ 0 & 1 & 3 & 1 \\ 0 & -1 & -2 & -3 \end{array} \right]$$

$$R_2 + R_3 \rightarrow R_3 \quad \left[ \begin{array}{ccc|c} 1 & 2 & 0 & 1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & -2 \end{array} \right] \quad R_2 - 3R_3 \rightarrow R_2 \quad \left[ \begin{array}{ccc|c} 1 & 2 & 0 & 1 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & -2 \end{array} \right]$$

$$R_1 - 2R_2 \rightarrow R_1 \quad \left[ \begin{array}{ccc|c} 1 & 0 & 0 & -13 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & -2 \end{array} \right] \quad \begin{aligned}x &= -13 \\y &= 7 \\z &= -2\end{aligned}$$

b) (12 pnt) Consider the linear system

$$x - z = 2$$

$$x + y - 3z = 3$$

$$-2y + 4z = -2$$

(i) Form the augmented matrix of the given system.

(ii) Solve the system by using matrix reduction.

$$\left[ \begin{array}{ccc|c} 1 & 0 & -1 & 2 \\ 1 & 1 & -3 & 3 \\ 0 & -2 & 4 & -2 \end{array} \right]$$

$$R_2 - R_1 \rightarrow R_2$$

$$\left[ \begin{array}{ccc|c} 1 & 0 & -1 & 2 \\ 0 & 1 & -2 & 1 \\ 0 & -2 & 4 & -2 \end{array} \right]$$

$$R_3 - 2R_2 \rightarrow R_3$$

$$\left[ \begin{array}{ccc|c} 1 & 0 & -1 & 2 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$x - z = 2$$

$$y - 2z = 1$$

$$z = t \Rightarrow x = 2 + t$$

$$y = 1 + 2t, \quad t \in \mathbb{R}$$

$$z = t$$